IS THE AUSTRALIAN FOREX MARKET EFFICIENT?

A TEST OF THE FORWARD RATE UNBIASEDNESS HYPOTHESIS

By

D.E. Allen
and
Paul Taco
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Correspondence author:
David E. Allen
School of Accounting, Finance and Economics
Faculty of Business and Law
Edith Cowan University
Joondalup, WA 6027
Australia
Phone: +618 6304 5471
Fax: +618 6304 5271
Email: d.allen@ecu.edu.au_

[1] School of Accounting, Finance and Economics, Edith Cowan University, Joondalup Drive, Joondalup. Tel 61863045471, Fax 61863045271, Email: d.allen@ecu.edu.au

Abstract

This paper features a test of the forward rate unbiasedness hypothesis (FRUH), using the Australian dollar with the United States and Japanese currencies using daily frequencies. We evaluate the FRUH on the 1-month forward rate, for both currencies, and the 3-month and 6-month forwards rates for the US dollar only. We adopt a cointegration framework for assessing the FRUH applying a cointegrating VAR model involving Johansen's ML approach. Our results indicate that in all cases the spot and forward rates are integrated of order 1. Furthermore there is evidence of cointegration and in all but one case the cointegrating vector is (1, -1). The error correction term in all cases is statistically significant and has the correct sign.

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1. **Introduction**

In this paper we test the theory of the Forward Rate Unbiasedness Hypothesis (FRUH); a theory that underpins much of the work done on the foreign exchange (FOREX) markets. These markets continually grow in significance; the Reserve Bank of Australia (RBA) reported that the size of the foreign exchange in Australia had grown to the seventh largest in the world by 2006. Some studies of exchange rates by Huang (1981), Vander Kraats and Booth (1983) and Wadhwani (1987) have suggested that exchange rates are 'too' volatile with respect to the behaviour of their underlying determinants. The issue of exchange rate volatility can be viewed as an efficient markets issue. "If exchange rates are too volatile with respect to a reasonable benchmark in an efficient market, then there may be grounds for throwing 'sand in the gears' of currency markets, so as to slow down changes in exchange rates and keep them in line with those of their 'fundamentals'" (Eichengreen, Tobin et al. 1995). "Conversely, if exchange rate volatility is largely consistent with that predicted by conventional models, then one may simply have to swallow the rates' abrupt swings as the potentially efficient response to underlying shocks" (Bartolini and Giorgianni 2001). Consequently this is a critical issue for policy makers, as exchange rate volatility has been linked to "growing trade imbalances, increased financial market volatility and less effective domestic macroeconomic policies" (Kahn 2000).

In this study we analyse the interest rate and exchange rate relationship, and explore how it affects the forward and spot rates of Australian denominated currency. We test whether the Australian Foreign Exchange is an efficient market in the context of a cointegrating relationship between forward rates and spot rates. Prior work in this area includes Hakkio and Rush (1989), Felmingham and Leong (2003) and Zivot (1998).

The paper is divided into six sections; a review of parity conditions follows in section 2, section three extends the theoretical framework and section four introduces the empirical methods adopted and the data set. Section five reviews the results and a brief conclusion follows in section six.

2. Interest parity conditions

Economic agents have a choice between "holding domestic-currency assets, which yield the own rate of interest r_d, or assets denominated in foreign currency, which yield the own rate of interest. To the extent that investors can accumulate either $(1+r_d)$ or $s(1+r_f)/f$ units of domestic currency with certainty, arbitrageurs in pursuit of assured profit will move funds in whatever amounts are required to eliminate any discrepancies between these interest factors. Thus an interest-rate parity condition is created, this condition asserts an asset-market equilibrium, where $(1+r_d) = s(1+r_f)/f$, which implies

$$(f-s)/s = (1+r_f)/(1+r_f)-1 = (r_f - r_d)/(1+r_d) \cong r_f - r_d \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

In other words, the percentage forward premium on domestic currency – i.e. the percentage by which the forward price of domestic currency exceeds the spot prices – will equilibrate to the excess of the foreign interest rate over the domestic interest rate" (Isard, 1978). Arbitrageurs should prevent large discrepancies between forward exchange rates and spot rate that are expected to prevail on the dates on which forward contracts matured.

Covered interest parity

An alternative way of expressing (1.1) is using a covered interest parity analysis, accounting for foreign exchange risk. This risk is that the future spot exchange rate may not equal the expected future spot rate. By purchasing a forward contract, the individual guarantees a rate of exchange at maturity, thus eliminating all foreign exchange uncertainty. Agents, who cover foreign exchange risk with forward contracts, should be indifferent between two different countries financial instruments.

Thus:

$$1 + r_d = (f/s)(1 + r_f)$$

$(f/s) = (s/s) + (f - s)/s = 1 + (f - s)/s$, rewriting

$1 + r_d = [1 + (f - s)/s](1 + r_f)$, simplifying

$$1 + r_d = 1 + (f - s)/s + r_f + r_f\,(f - s)/s$$

because r_f and $(f - s)/s$ are both typically small fractions, their product is approx. 0, thus simplifying

$$r_d - r_f \cong \frac{f - s}{s}$$

Again equation 1.1 is reproduced, with an important implication that introduces the partnering interest rate condition of Uncovered interest parity (UIP).

Uncovered interest parity

Contrary to CIP, there is no attempt to hedge foreign exchanger risk using forward contracts, leaving foreign transactions uncovered and hinging on expectations of the future spot rate. This creates the uncovered interest parity (UIP), (Daniels and VanHoose 2001). This can be summarised as:

$$r_d - r_f \cong \frac{s_{+1}^e - s}{s} \quad \text{...(2)}$$

Furthermore, because the transaction is uncovered, there is no effect on the forward market, the effect on the next period's spot exchange rate $\left(s_{+1}^e\right)$, would be the same as on the forward market in CIP. It is evident that the two parity conditions are linked through interest rate differentials, "suppose that both conditions are satisfied, thus:

$$\frac{f - s}{s} \cong r_d - r_f \cong \frac{s_{+1}^e - s}{s} \quad \text{..(3)}$$

These conditions imply that the forward premium is equal to the change in the spot rate, and simplifying we obtain the following condition:

$$f = s_{+1}^e \quad \text{..(4)}$$

This signals that the forward exchange rate should equal the spot rate expected to prevail at the time of the settlement of the forward contract. If the forward rate systematically differs from the spot rate, then in the absence of a risk premium, a profit opportunity exists.

The implication of combining an efficient market with condition (4) is that the forward exchange rate should on average equal the expected future spot exchange rate. In other words, the forward exchange rate is an unbiased predictor of the expected future spot rate. In the following analysis “If the forward exchange rate for t+1 is an unbiased rational expectation of the rate at t+1, cointegration of the forward and future spot rate follows simply” (Dwyer and Wallace 1992).

3. The theoretical framework

This study analyses the forward rate unbiasedness hypothesis using the Australian dollar with the United States dollar and the Japanese yen using a cointegration framework. Japan is currently Australia’s largest trading partner and the United-States is Australia’s third largest trading partner. We eschewed the use of China given that its currency is managed to a degree and not subject to the same market forces.

There are numerous previous Studies testing the FRUH; Engle (1996) provides a review, nevertheless only a few have analyzed the relationship between spot and forward rates using Australian data. The most recent study done was by Felmingham and Leong (2003) who used Australian and US daily data for 90 and 180-day forward markets for the period of 1985 to 2000. Felmingham and Leong (2003) indicate that forward rates acted as an unbiased predictor of the spot rate for the period 1992 –2000 using AUD/USD data but only in the 90-day market. However, when they consider the cash rate simultaneously, the forward rate in the 180-day market is also unbiased. Thus they conclude that the predictability of the spot exchange rate through the forward rate has improved over time.

In this study, we will examine both exchange rates using Australia as the denominated currency. Our data will be daily and the most recent, for example the AUD/USD spans the last three years, from Jan 03 to Dec 06. Furthermore, due to the growth in cointegrating VAR analysis and the framework provided by Zivot (1998) we have new methods for testing the FRUH, which will be applied in this paper.

Hakkio and Rush (1989) establish that historically there have been two general approaches to testing the forward rate unbiasedness hypothesis (FRUH); the first is the 'levels regression'. In this regression, researches regress the future sport rate, s_{t+1}, on the forward rate, f_t, f_t, as in equation (5).

$$s_{t+1} = a + bf_t + e_t \quad \text{...(5)}$$

Early researches like Frenkel (1976, 1979) found that *b* estimates were very close to 1 and hence supported the FRUH. However subsequent authors such as Meese and Singleton (1982), have criticised this approach because of the potential non-stationarity of the spot and forward rates, where by both series follow unit root processes and this implies that spot and forward rates are cointegration. This implies potential spurious regression problems as described in Granger and Newbold (1974). The second approach to testing efficiency is the 'difference equation', this method regresses the rate of depreciation on the forward premium, as in equation (6)

$$\Delta s_{t+1} = a + b[f_t - s_t] + u_t \quad \text{...................................... (6)}$$

Empirical studies such as Bilson (1981), Hansen and Hodrick (1980) and Huang (1981) have found that under this approach the FRUH was overwhelmingly rejected. Furthermore typical results of the estimate *b* across a wide range of currencies and sampling frequencies are found to be significantly negative. These results are therefore referred to as the 'forward discount anomaly', 'forward discount bias' or 'forward discount puzzle' and seem to contradict the results based on the 'levels regression'.

Hakkio (1981) and Baillie, Lippens, and McMahon (1983) in the light of these deficiencies and new developments in the theory of cointegration by Engle and Granger (1987) formed a third approach. Market efficiency implies that even if the spot and forward rate are non-stationary, they never drift apart so that they will be cointegrated" (Hakkio and Rush 1989). The cointegrating relationship between spot and forward rates, in either the 'levels' or 'differences' approach, has been examined by many including Hakkio and Rush (1989), Barhart and Szakmary (1991), Naka and Whitney (1995), Hai, Mark and Yu (1997), and Clarida and Taylor (1997). However results from these studies were mixed and strongly dependent on the how cointegration relationship was modelled.

An important insight from all the cointegration studies was the recognition that the FRUH requires that s_{t+1} and f_t or f_t and s_t be cointegrated with a cointegrating vector of (1,-1). Moreover Zivot (1997) points out that since:

$$s_{t+1} - f_t = \Delta s_{t+1} - (f_t - s_t)$$

"It is trivial to see under the assumption that f_t and s_t are I(1) that (i) if s_t and f_t are cointegrated with cointegrating vector (1,-1) then s_{t+1} and f_t must be cointegrated with cointegrating vector (1,-1); and (ii) if s_{t+1} and f_t are cointegrated with cointegrated vector (1,-1) then s_t and f_t must be cointegrated with cointegrating vector (1,-1)" (Zivot 1997). However simple first order vector error correction models that use (s_{t+1}, f_t) must be used with caution, as they miss some important dynamics in monthly data and as a result indicate that the FRUH appears to hold.

4 The Research Method and data set

We commence by undertaking unit root tests to determine whether the series are non-stationary using the Augmented Dickey-Fuller (ADF) tests recommended by Engle and Granger (1987). We then proceed to capture dynamics using a simple cointegrated VAR(1) model for $y_t = (f_t, s_t)$ and thus inferences from this model are true and reliable. We follow Zivot (1998) writing a bivariate VAR(1) model for y_t, which is:

$$y_t = \mu + \Phi y_{t-1} + \in_t \text{……………………………………..(7)}$$

$$y_t = A' g_t + \in_t \text{ ……………………………………………(8)}$$

where y_t is an $m \times 1$ vector of jointly determined (endogenous) variables, and $\in_t$ is an $m \times 1$ vector of unobserved disturbances assumed to satisfy the following assumptions:

Having established VAR(1) model, we can also write the system of equations in (7) as a SURE model (8) with all the equations having the same set of regressors, $g_t = (y_{t-1}, y_{t-2}, \dots, y_{t-p})$, in common. This is done to calculate the Maximum Likelihood (ML) estimators of the unknown coefficients when regressing y_t on g_t and further to calculate the VAR order selection. Firstly we must write (8) in matrix notation to have

$$\underset{n \times m}{Y} = \underset{n \times s}{G} \underset{s \times m}{A} + \underset{n \times m}{\in} \text{ ……………………………………..(9)}$$

where: $s = mp + 2$;

$$\underset{n\times m}{Y} = (y_1, y_2, \ldots, y_n)' ;$$

$$\underset{m\times s}{A} = (\mu, \Phi);$$

$\underset{n\times(mp+2)}{g} = (\iota_n, t_n, Y_{-1}, Y_{-2}, \ldots, Y_{-p})$, where ι_n and t_n are the n-dimensional vectors (1,1,…,1)' and (1,2,…,n), respectively.

The ML estimators of A and Σ are given by

$$\hat{A} = (G'G)^{-1}G'Z \quad \text{..(10)}$$

and

$$\tilde{\Sigma} = n^{-1}(Y - G\hat{A})'(Y - G\hat{A}) \quad \text{..(11)}$$

These equations will be used when selecting the VAR order selection criterion, in particular when using the Akaike Information Criterion (AIC) and the Schwarz Bayesian Criterion (SBC); Returning to the our VAR(1) model, and using the analysis from (Zivot, 1998) we calculate the cointegrating VAR model, by rewriting equation (7) as

$$\Delta y_t = \mu + \prod y_{t-1} + \in_t \quad \text{.(12)}$$

where $\prod = \Phi - I$, we are able to summarize the long-run information in y_t by the long-run impact matrix, $\prod$; it is the rank of this matrix that determines the number of cointegrating vectors. In our case the rank is 1 and there exists 2×1

vectors α and β such that $\prod = \alpha\beta'$. Using the normalization $\beta = (1 - \beta_s)'$, (2.7) becomes the vector error correction model (VECM):

$$\Delta f_t = \mu_f + \alpha_f \left(f_{t-1} - \beta_s s_{t-1}\right) + \in_{ft} \quad \text{.................................(12a)}$$

$$\Delta s_t = \mu_s + \alpha_s \left(f_{t-1} - \beta_s s_{t-1}\right) + \in_{st} \quad \text{..................................(12b)}$$

Since spot and forward rates do not exhibit a systematic tendency to drift up or down it may be more appropriate to restrict the intercepts in (12) to the error correction term. That is, $\mu_f = -\alpha_f \mu_c$ and $\mu_s = -\alpha_s \mu_c$. Under this restriction s_t and f_t are I(1) without drift and the cointegrating residual, $f_t - \beta_s s_t$, is allowed to have a nonzero mean μ_c.

With the intercepts in (12) restricted to the error correction term, the VECM can be solved to give a simple AR(1) model for the cointegrating residual $\beta' y_t - \mu_c = f_t - \beta_s s_t - \mu_c$. Pre-multiplying (9) by β' and rearranging gives:

$$f_t - \beta_s s_t - \mu_c = \Phi\left(f_{t-1} - \beta_s s_{t-1} - \mu_c\right) + \eta_t \quad \text{.....................(13)}$$

where $\Phi = I + \beta'\alpha = I + \left(\alpha_f - \beta_s \alpha_s\right)$ and $\eta_t = \beta' \in_t = \in_{ft} - \beta_s \in_{st}$. Since (13) is simply an AR (1) model, the cointegrating residual is stable and stationary if $|\Phi| = \left|1 + \left(\alpha_f - \beta_s \alpha_s\right)\right|$ < 1. Notice that if $a_f = \beta_s a_s$ then the cointegrating residual is *I*(1) thus f_t and s_t are not cointegrated.

Another important issue with this model is the exogeneity status of spot and forward rates with regards to the cointegrating parameters α and β. This was the focus of many authors, who were concerned with exogeneity issues in error correction models, such as Johansen (1992, 1995) and Zivot (1998). In this model, exogeneity of spot and forward rates places restrictions on the parameters of the VECM (12), in particular if f_t is weakly exogenous with respect to $(\alpha_s \beta_s)'$. If this is the case then $\alpha_f = 0$ and efficient estimation of the cointegrating parameters can be made from the single equation conditional error correction model,

$$\Delta s_t = \mu_s + \alpha_s (f_{t-1} - \beta_s s_{t-1}) + \gamma_s \Delta f_t + \nu_{st} \quad \text{..........................(13a)}$$

where $\gamma_s = \sigma_{ss}^{-1} \sigma_{fs}$ and ν_{st} is uncorrelated with $\in_{ft}$.

This is also the case if s_t is weakly exogenous with respect to $(\alpha_f \beta_s)'$ then $\alpha_s = 0$ and efficient estimation of the cointegrating parameters can be made from the single equation conditional error correction model,

$$\Delta f_t = \mu_f + \alpha_f (f_{t-1} - \beta_s s_{t-1}) + \gamma_j \Delta s_t + \nu_{ft} \quad \text{........................(13b)}$$

where $\gamma_f = \sigma_{ff}^{-1} \sigma_{fs}$ and ν_{ft} is uncorrelated with $\in_{st}$.

If $\beta_s = 1$ then the forward premium is $I(0)$ and follows an AR(1) process and the VECM (12) becomes,

$$\Delta f_t = \mu_f + \alpha_f (f_{t-1} - s_{t-1}) + \in_{ft}, \quad \text{.................................(14a)}$$

$$\Delta s_t = \mu_s + \alpha_s (f_{t-1} - s_{t-1}) + \in_{st} \quad \text{.................................(14b)}$$

Notice that (14b) is simply the standard differences regression used to test the FRUH. Furthermore, if α_j and α_s are of the same sign and magnitude then the implied value of Φ in (13) is close to 1 and this corresponds to the stylized facts described above, in particular where the forward premium is stationary but very highly autocorrelated. Also, the implied variance of the forward premium from (12) is $\sigma_{\eta\eta} = \sigma_{ff} + \sigma_{ss} - 2\rho_{fs}\left(\sigma_{ff}\sigma_{ss}\right)^{1/2}$ and will be very small relative to the variances of Δf_t and Δs_t given the stylized facts that $\sigma_{ff} \approx \sigma_{ss}$ and $\rho_{fs} \approx 1$.

The FRUH places testable restrictions on the VECM (). Necessary conditions for the FRUH to hold are (i) s_t and f_t are cointegrated (ii) $\beta_s = 1$ and (iii) $\mu_c = 0$. In addition, the FRUH requires that $\alpha_s = 1$ in order for the forecast error to have conditional mean zero. Together these two restrictions limit both the long-run and short-run behaviour of spot and forward rates. Applying these restrictions, (12), one period ahead, becomes,

$$\Delta f_{t+1} = \alpha_f\left(f_t - s_t\right) + \in_{f,t+1} \quad \text{(15a)}$$

$$\Delta s_{t+1} = \left(f_t - s_t\right) + \in_{s,t+1} \quad \text{(15b)}$$

Notice that the FRUH requires that the expected change spot rate is equal to the forward premium or, equivalently, that the adjustment to long-run equilibrium occurs in one period. The change in the forward rate, on the other hand, is directly related to the persistence of interest rate differentials now measured by α_f since $\Phi = I + \left(\alpha_f - I\right) = \alpha_f$. Stability of the VECM under the FRUH requires that $\left|\alpha_f\right| < 1$. Thus, the FRUH is consistent with a highly persistent forward premium.

The representation in (15) shows that weak exogeneity of spot rates with respect to the cointegrating parameters is inconsistent with the FRUH because if spot rates are weakly exogenous then $\alpha_s = 0$ and the FRUH cannot hold. In addition if the FRUH is true and forward rates are weakly exogenous then (15) cannot capture the dynamics of typical data. To see this, suppose that forward rates are weakly exogenous so that $\alpha_f = 0$. Since $\sigma_{ss} \approx \sigma_{ff} \approx \sigma_{sf} = \sigma^2$ it follows that $\gamma_s \approx \gamma_f \approx 1$. If $\mu_s = 0$, $\alpha_s = 1$ and $\beta_s = 1$, then (13) becomes

$$s_t = f_{t-1} + \Delta f_t + \nu_t = f_t + \nu_t$$

which simply states that the current spot rate is equal to the current forward rate plus a white noise error.

We proceed by performing the ADF tests, then test the hypothesis of whether the realized spot and forward rates are cointegrated using a vector autoregression (VAR) model. We first run an unrestricted VAR on Spot and forward rates, in order to select and determine the order of VAR. Using the Akaike Information Criterion (AIC) and Schwarz Bayesian Criterion (SBC), and then ascertain the number of cointegrating relationships;

Our prior is one cointegrating relation, and we can impose it on the model, by specifying *r* (the number of cointegrating vectors). This then allows us to start our long-run structural modelling analysis, Pesaran and Smith (1996), in particular the testing of the cointegrating vector of (1,-1), which has been extensively documented in previous studies.

Data set

We use daily data from January 2003 to December 2006. The daily data for AUD/JPY will be from July 06 to December 06. The forward markets used for AUD/USD analysis was the 1-month forward, 3-month forward, and 6-month forward rates, whilst the AUD/JPY analysis used on the 1-month forward rates. All AUD/JPY data was obtained from Datastream, whilst all AUD/USD data was obtained from the Reserve Bank of Australia (RBA).

5. Results

We conducted ADF tests, to examine whether the following spot or forward rates are stationary or non-stationary. From the results in Table 1 it is evident that the AUD/USD Spot and 1-month forward are non-stationary, indeed all variables confirm the necessary condition for cointegration analysis. We conducted tests to select the order of VAR. In Table 2, where we evaluate the AUD/USD Spot and 1-month forward rate, we can see the highest value for AIC is 7067.3 and for SBC is 7052.6 which both correspond to a VAR of the order of 1. This is also confirmed in table 3 and 4, where we consider the AUD/USD 3-month and 6-month forward rates. Evaluating Table 5, the AUD/JPY Spot and 1-month forward rate, we also come to the same conclusion where the AIC and SBC both select a VAR order of 1. This means that our model will be VAR(1) model, which is consistent with work by Zivot (1997).

Table 1: Augmented Dickey Fuller (ADF) Tests for AUD/USD Spot and All Forward Rates

Variables	ADF Test Statistic	ADF 95% Critical Value	Significant at 5% level
		Actual Exchange Rates	
Spot Rate	-3.0343	-3.4167	No
1-Month Forward Rate	-2.6736	-3.4167	No
3-Month Forward Rate	-2.3029	-3.4167	No
6-Month Forward Rate	-1.5579	-3.4167	No
		1st Difference Exchange Rates	
Spot Rate	-22.0911	-3.4167	Yes
1-Month Forward Rate	-22.1762	-3.4167	Yes
3-Month Forward Rate	-22.3374	-3.4167	Yes
6-Month Forward Rate	-22.5453	-3.4167	Yes

Note: This table displays the ADF test statistic in absolute value, where the number of lags for each test is 1. Furthermore the 95% critical values, the null hypothesis, of a unit root, against the alternative, of no unit root. If a variable has a non-significant test result at a 5% level of significance, it contains a unit root and thus is non-stationary or integrated.

Table 8: ADF Tests for AUD/JPY Spot and 1-Month Forward Rates

Variables	ADF Test Statistic	ADF 95% Critical Value	Significant at 5% level
		Actual Exchange Rates	
Spot Rate	-3.3870	-3.4519	No
1-Month Forward Rate	-2.5385	-3.4519	No
		1st Difference Exchange Rates	
Spot Rate	-8.0385	-3.4523	Yes
1-Month Forward Rate	-8.0085	-3.4523	Yes

Table 2: Unrestricted VAR analysis – AUD/USD Spot Rate and 1-Month Forward Rate

Order	LL	AIC	SBC	LR Test
4	7081.7	7063.7	7019.7	--------------
3	7078.2	7064.2	7030.0	CHSQ (4) = 6.9670[.138]
2	7076.8	7066.8	7042.4	CHSQ (8) = 9.8379[.277]
1	7073.3	7067.3*	7052.6*	CHSQ (12) = 16.8934[.154]
0	3229.1	3227.1	3222.2	CHSQ (16) = 7705.3[.000]

AIC=Akaike Information Criterion SBC=Schwarz Bayesian Criterion

Note: These tables are used to assess the lag length in this cointegrating VAR model; firstly a hypothesised lag order of 1 to 4 is selected. The resulting AIC and SBC model selection criteria are computed for each corresponding lag length. The order of VAR [VAR(*p*)] selected corresponds with the highest value of the SBC and AIC. The highest AIC and SBC is denoted by *.

Table 3: Unrestricted VAR analysis – AUD/USD Spot Rate and 3-Month Forward Rate

Order	LL	AIC	SBC	LR Test
4	7058.1	7040.1	6996.1	--------------
3	7054.8	7040.8	7006.6	CHSQ (4) = 6.5121[.164]
2	7054.2	7044.2	7019.8	CHSQ (8) = 7.7022[.463]
1	7052.8	7046.8*	7032.1*	CHSQ (12) = 10.5569[.567]
0	2715.7	2713.7	2708.8	CHSQ (16) = 8684.8[.000]

AIC=Akaike Information Criterion SBC=Schwarz Bayesian Criterion

Table 4: Unrestricted VAR Analysis – AUD/USD Spot Rate and 6-Month Forward Rate

Order	LL	AIC	SBC	LR Test
4	7033.1	7015.1	6971.1	--------------
3	7030.1	7016.1	6981.8	CHSQ (4) = 6.0214[.198]
2	7029.1	7019.1	6994.7	CHSQ (8) = 7.8766[.446]
1	7028.3	7022.3*	7007.6*	CHSQ (12) = 9.6415[.647]
0	2326.0	2324.0	2319.1	CHSQ (16) = 9414.2[.000]

AIC=Akaike Information Criterion SBC=Schwarz Bayesian Criterion

Table 5: Unrestricted VAR Analysis – AUD/JPY Spot and 1-Month Forward Rate

Order	LL	AIC	SBC	LR Test
4	872.7221	856.7221	835.4904	--------------
3	870.3805	858.3805	842.4568	CHSQ (4) = 4.6831[.321]
2	868.4452	860.4452	849.8293	CHSQ (8) = 8.5538[.381]
1	867.5564	863.5564*	858.2485*	CHSQ (12) = 10.3314[.587]
0	-16.5212	-16.5212	-16.5212	CHSQ (16) = 1778.5[.000]

AIC=Akaike Information Criterion SBC=Schwarz Bayesian Criterion

Cointegration of spot and forward rates.

Once the order of VAR is set to 1, we can then determine how many cointegrating relationships are present between Spot and the forward rates. Logic suggests that its is unlikely that a trend exists in the cointegrating relationship between spot and forward rates, thus the 'restricted intercepts and no trends' option was chosen for this model.

The results in table 6 show that both the Maximal eigenvalue and Trace statistic strongly reject the hypothesis that there exists no cointegration. However both tests affirm that there exist more than one cointegration relationship. Conversely looking at the Model Selection Criteria only Schwarz Bayesian Criterion, suggest that there exists 1 cointegrating relationship, and the others suggest more than one. The results in table 7 for the three month forward rates are similar, with only the SBC suggesting one cointegrating relationship whilst the other tests all suggest two or more. In Table 8, the Spot and 6-month forward rates are the only AUD/USD test that affirms one cointegrating relationship, however not collectively. As only the Trace statistic rejects $r = 0$ (no cointegration) and affirms $r = 1$ (one cointegration relation), but the Maximal Eigenvalue does not reject $r = 0$ and thus suggests no cointegration exists between the Spot and 6-month forward rate. Now viewing Model Selection Criteria, we again see that only the SBC supports the Trace statistic findings, whilst the other criteria support either 'no-cointegration' or 'two or more cointegrating relations'. Finally in table 9, the AUD/JPY Spot and 1-month forward rates, show inconclusive results. Both the Maximal Eigenvalue and Trace Statistic show that there exists no cointegration (r = 0) at both the 90% and 95% level of significance. Furthermore only the SBC in the Model Selection Criteria supports the hypothesis of $r = 1$.

Table 6: Cointegration of AUD/USD Spot and 1-Month Forward Rates

Cointegration LR Test based on Maximal Eigenvalue				
Null	*Alternative*	*Statistic*	*95% Critical Value*	*90% Critical Value*
r = 0	r = 1	50.0065	15.8700	13.8100
r <= 1	r = 2	12.9868	9.1600	7.5300
Cointegration LR Test based on Trace				
Null	*Alternative*	*Statistic*	*95% Critical Value*	*90% Critical Value*
r = 0	r = 1	62.9934	20.1800	17.8800
r <= 1	r = 2	12.9868	9.1600	7.5300
Choice of the Number of Cointegrating Relations Using Model Selection Criteria				
Rank	*Maximized LL*	*AIC*	*SBC*	*HQC*
r = 0	7064.5	7064.5	7064.5	7064.5
r = 1	7089.5	7085.5	7075.7*	7081.8
r = 2	7096.0*	7090.0*	7075.3	7084.4*

AIC = Akaike Information Criterion   SBC = Schwarz Bayesian Criterion
HQC = Hannan-Quinn Criterion

Notes[1]: This table displays the following tests and model selection criteria that is used to determine the appropriate number of cointegrating relations that are likely to exist among the *I*(1) variables.

Notes[2]: The tests computed are the Trace and Maximum Eigenvalue statistics, they are used for testing the rank of the long-run matrix, Πy, as 0, 1, or 2 together with the relevant 90% and 95% critical values. Any significant values at a 90% level will be denotes as *, and any significant values at a 95% level will be denoted as **.

Notes[3]: This table further presents the maximized values of the log-likelihood function of the cointegrating VAR model, Akaike, Schwarz, and Hannan and Quinn model selection criteria, for the different values of r, the rank of the long run matrix, Πy. The highest value for each criteria is displayed as *.

Table 7: Cointegration of AUD/USD Spot and 3-Month Forward Rates

Cointegration LR Test based on Maximal Eigenvalue				
Null	*Alternative*	*Statistic*	*95% Critical Value*	*90% Critical Value*
r = 0	r = 1	16.5069	15.8700	13.8100
r <= 1	r = 2	12.5901	9.1600	7.5300
Cointegration LR Test based on Trace				
Null	*Alternative*	*Statistic*	*95% Critical Value*	*90% Critical Value*
r = 0	r = 1	29.0971	20.1800	17.8800
r <= 1	r = 2	12.5901	9.1600	7.5300
Choice of the Number of Cointegrating Relations Using Model Selection Criteria				
Rank	*Maximized LL*	*AIC*	*SBC*	*HQC*
r = 0	7060.8	7060.8	7060.8	7060.8
r = 1	7069.1	7065.1	7055.3*	7061.4
r = 2	7075.4*	7069.4*	7054.7	7063.8*

AIC = Akaike Information Criterion   SBC = Schwarz Bayesian Criterion
HQC = Hannan-Quinn Criterion

Table 8: Cointegration of AUD/USD Spot and 6-Month Forward Rates

Cointegration LR Test based on Maximal Eigenvalue				
Null	*Alternative*	*Statistic*	*95% Critical Value*	*90% Critical Value*
r = 0	r = 1	13.3575	15.8700	13.8100
r <= 1	r = 2	8.2359	9.1600	7.5300

Cointegration LR Test based on Trace				
Null	*Alternative*	*Statistic*	*95% Critical Value*	*90% Critical Value*
r = 0	r = 1	21.5933	20.1800	17.8800
r <= 1	r = 2	8.2359*	9.1600	7.5300

Choice of the Number of Cointegrating Relations Using Model Selection Criteria				
Rank	*Maximized LL*	*AIC*	*SBC*	*HQC*
r = 0	7039.8	7039.8	7039.8*	7039.8*
r = 1	7046.5	7042.5	7032.7	7038.7
r = 2	7050.6*	7044.6*	7029.9	7039.0

AIC = Akaike Information Criterion   SBC = Schwarz Bayesian Criterion
HQC = Hannan-Quinn Criterion

Table 9: Cointegration of AUD/JPY Spot and 1-Month Forward Rates

Cointegration LR Test based on Maximal Eigenvalue				
Null	*Alternative*	*Statistic*	*95% Critical Value*	*90% Critical Value*
r = 0	r = 1	8.8147	15.8700	13.8100
r <= 1	r = 2	5.7011	9.1600	7.5300

Cointegration LR Test based on Trace				
Null	*Alternative*	*Statistic*	*95% Critical Value*	*90% Critical Value*
r = 0	r = 1	14.5157	20.1800	17.8800
r <= 1	r = 2	5.7011	9.1600	7.5300

Choice of the Number of Cointegrating Relations Using Model Selection Criteria				
Rank	*Maximized LL*	*AIC*	*SBC*	*HQC*
r = 0	882.7123	882.7123	882.7123*	882.7123*
r = 1	887.1196	883.1196	877.7554	880.9446
r = 2	889.9702*	883.9702*	875.9238	880.7076

AIC = Akaike Information Criterion   SBC = Schwarz Bayesian Criterion
HQC = Hannan-Quinn Criterion

Theses results are puzzling and difficult to square with existing literature on FRUH. Moreover, there is a general consensus that there exists 1 cointegrating relationship between spot and forward rates, combining this with the mixed results from short data samples, we will impose $r = 1$ on the model. The following estimates of the cointegrating coefficients are normalized on the coefficient of the spot exchange rate. This is achieved by placing the restriction of A1 = 1 to the model, which is the Spot rate or the first variable in the cointegrating VAR. In the following tables we display the estimates of the forward rates and their asymptotic standard errors in Panel A.

Looking at table 10, we can see that the 1-month forward rate is negative and very close to one, which according to preceding literature and economic theory is precisely what it should be, thus imposing the further restriction of [A2 = -1] should only strengthen this relationship. Panel B in table 11 displays this further restriction, and looking at the log-likelihood ratio statistic for testing this restriction, which is 0.0052 [0.943], confirms that this restriction cannot be rejected, as it is not statistically significant and thus affirms this relationship.

Table 11 panel A shows that the 3-month forward rate is negative and just over one, which again is the highly reported (1, -1) cointegrating relationship. By imposing the further exact restriction in panel B, we obtain the following log-likelihood ratio statistic 0.86177(0.353), which again is not significant and thus this relationship cannot be rejected.

Table 12: panel (A), displays the 6-month forward rate as a very small negative, which is the only AUD/USD rate not close to one. Imposing the further restriction displayed in panel B, shows the following log-likelihood ratio statistic 4.9489(.026), which is statistically significant at a 95% level, and thus provides evidence against the validity of the cointegrating relation (1, -1). Table 13: panel (A), shows the 1-month forward rate as positive however not large number, again contradictory to the cointegrating relation (1, -1). Imposing the over-identifying restriction of A2=-1, displayed in panel B, we obtain a non-significant log-likelihood ratio statistic. This affirms the cointegration relation between spot and forward rates.

Table 10: Restrictions on AUD/USD Spot and 1-Month Forward Rates

Panel A	A1 = -1
Spot	1.0000 (*None*)
1-Month Forward	-0.99562 (0.060559)
Panel B	A1 = -1; A2 = 1
Spot	1.0000 (*None*)
1-Month Forward	-1.0000 (*None*)
LL Subject to exactly identifying LR tests of Restrictions	CHSQ (1) = 0.0051780 [0.943]

Notes[1]: Panel A displays the estimates of the VAR model subject to the general restriction on the contegrating coefficient, in this case the coefficient of the Spot rate is normalized to 1, the first variable in the contegrating VAR. Further the estimate of the cointegrating coefficient (1-Month Forward rate) is displayed along with its asymptotic standard errors.

Notes2: Panel B displays the imposed over-identifying restriction of A2=-1, where A2 stands for the coefficient of the 1-Month Forward rate. This restriction is tested using a 'log-likelihood ratio statistic' denoted by CHSQ (1).

Table 11: Restrictions on AUD/USD Spot and 3-Month Forward Rates

Panel A	A1 = -1
Spot	1.0000 (*None*)
1-Month Forward	-1.7572 (2.0072)
Panel B	A1 = -1; A2 = 1
Spot	1.0000 (*None*)
1-Month Forward	-1.0000 (*None*)
LL Subject to exactly identifying LR tests of Restrictions	CHSQ (1) = 0.86177 [.353]

Table 12: Restrictions on AUD/USD Spot and 6-Month Forward Rates

Panel A	A1 = -1
Spot	1.0000 (*None*)
1-Month Forward	-0.24484 (0.26306)
Panel B	A1 = -1; A2 = 1
Spot	1.0000 (*None*)
1-Month Forward	-1.0000 (*None*)
LL Subject to exactly identifying LR tests of Restrictions	CHSQ (1) = 4.9489 [.026]

Table 13: Restrictions on AUD/JPY Spot and 1-Month Forward Rates

Panel A	A1 = -1
Spot	1.0000 (*None*)
1-Month Forward	2.7465 (19.6867)
Panel B	A1 = -1; A2 = 1
Spot	1.0000 (*None*)
1-Month Forward	-1.0000 (*None*)
LL Subject to exactly identifying LR tests of Restrictions	CHSQ (1) = 0.99504 [.319]

Error Correction Model (ECM) estimated by OLS based on Cointegrating VAR (1) Model

In this section, we assess the error correction form of the relation in the cointegrating VAR model. The results displayed in table 14, show the error correction equations and their corresponding test statistics for each AUD/USD 1-month, 3-month and 6-month forward rates.

In panel A, we can see that the error-correction term, 0.050344 (0.0070339) is statistically significant at the 95% level, thus reinforcing the existence of a long-run relationship between Spot and 1-month forward rate. The EC term has the correct negative sign, but it's very small, suggesting that it converges very slowly to equilibrium, with only 5.03% of the discrepancy corrected for each period. Further the goodness of fit statistic is 0.047148, indicating that the model does not explain the relationship well. From panel B, we see that the error-correction term, 0.015447 (0.0038922) is again statistically significant, and again supports the existence of a long-run relationship between the Spot and 3-month forward rate. The EC term has the correct sign, and again is very small. Finally from panel C, we see that the error-correction term, 0.0066716 (0.0023545), is significant, again supporting the long-run relationship between the Spot and 6-month forward rate. The EC term has the correct sign, but again is extremely small.

Similarly in Table 15, we see that the error-correction term, 0.04134 (0.01481), is significant at the 95% level, suggesting a long-run relationship between the AUD/JPY Spot and 1-month forward rate and the EC term has the correct sign, but again is very small.

Table 14: Error Correction Model for AUD/USD Spot and Forward Rates

Panel A	
The Error correction model	$\Delta Sp_t = \beta \Delta Fwd_t + \beta EC_{t-1}$
The Est. Model	ΔSpot = -1(Forward Rate) + 0.050334(EC_{t-1})
Coefficient	0.050344
Standard Error	0.0070339
T-Ratio [Prob.]	7.1574 [.000]
R-Squared	0.047148
R-Bar-Squared	0.047148
DW Statistic	2.0224
Diagnostic Tests	Test statistic [*p*-value]
A: *Serial Correlation*	CHSQ (1) = 0.13617 [.712]
B: *Functional Form*	CHSQ (1) = 0.6235E-4 [.994]
C: *Normality*	CHSQ (2) = 52.3466 [.000]
D: *Heter-oscedasticity*	CHSQ (1) = 9.9432 [.002]

Notes[1]: The diagnostic results utilize the following tests:
A: Lagrange multiplier test of residual serial correlation
B: Ramsey's RESET test using the square of the fitted values
C: Based on a test of skewness and kurtosis of residuals
D: Based on the regression of squared residuals on squared fitted values

Panel B	
The Error correction model	$\Delta Sp_t = \beta \Delta Fwd_t + \beta EC_{t-1}$
The Est. Model	ΔSpot = -1(Forward Rate) + 0.050334(EC_{t-1})
Coefficient	0.015447
Standard Error	0.0038922
T-Ratio [Prob.]	3.9688 [.000]
R-Squared	0.012990
R-Bar-Squared	0.012990
DW Statistic	2.0232
Diagnostic Tests	Test statistic [*p*-value]
A: *Serial Correlation*	CHSQ (1) = 0.13734[.711]
B: *Functional Form*	CHSQ (1) = 0.11090[.739]
C: *Normality*	CHSQ (2) = 98.4009[.000]
D: *Heter-oscedasticity*	CHSQ (1) = 0.28460 [.594]

Panel C	
The Error correction model	$\Delta Sp_t = \beta \Delta Fwd_t + \beta EC_{t-1}$
The Est. Model	ΔSpot = -1(Forward Rate) + 0.050334(EC_{t-1})
Coefficient	0.0066716
Standard Error	0.0023545
T-Ratio [Prob.]	2.8336 [.005]
R-Squared	0.0060485
R-Bar-Squared	0.0060485
DW Statistic	2.0232
Diagnostic Tests	Test statistic [*p*-value]
A: *Serial Correlation*	CHSQ (1) = 0.14852 [.700]
B: *Functional Form*	CHSQ (1) = 1.6043 [.205]
C: *Normality*	CHSQ (2) = 110.312 [.000]
D: *Heter-oscedasticity*	CHSQ (1) = 0.29750 [.585]

Table 15: Error Correction Model for AUD/JPY Spot and 1-Month Forward Rate

The Error correction model	$\Delta Sp_t = \beta \Delta Fwd_t + \beta EC_{t-1}$
The Est. Model	ΔSpot = -1(Forward Rate) + 0.050334(EC_{t-1})
Coefficient	0.040134
Standard Error	0.014810
T-Ratio [Prob.]	2.7100 [0.008]
R-Squared	0.032071
R-Bar-Squared	0.032071
DW Statistic	1.8126
Diagnostic Tests	Test statistic [*p*-value]
A: *Serial Correlation*	CHSQ (1) = 0.87798 [0.349]
B: *Functional Form*	CHSQ (1) = 0.18423 [0.668]
C: *Normality*	CHSQ (2) = 2.1132 [0.348]
D: *Heter-oscedasticity*	CHSQ (1) = 0.0025804 [0.959]

Notes1: The diagnostic results utilize the following tests:
A: Lagrange multiplier test of residual serial correlation
B: Ramsey's RESET test using the square of the fitted values
C: Based on a test of skewness and kurtosis of residuals
D: Based on the regression of squared residuals on squared fitted values

6. Conclusion

On balance our results seem to support the forward rate unbiasedness hypothesis (FRUH). As both exchange rates, the AUD/USD and the AUD/JPY, exhibit a cointegrating relationship between spot and forward rates, and furthermore our long-run structural modelling tests do not reject the cointegrating vector being (1, -1).

We began by testing the stationarity of the spot and forward rates for both exchange rates and the results suggests the spot and forward rates are integrated of order 1, I(1). We then assessed the order of VAR for this model, unanimously confirming the use of VAR(1) model. Next, we tested for the number of cointegrating relations present. These were somewhat inconsistent with different tests suggesting different numbers of cointegrating relations; ranging from no-cointegration present, to 1 cointegrating relation, and most commonly 2 or more. However we appealed to with economic theory and imposed 1 cointegrating relationship on the model.

Thus, we then imposed restrictions on the model and appealed to the data set to see if it rejected the restriction of a cointegrating relationship of the vector (1, -1). The results from the restrictions applied on both exchange rates were in favour of the this restriction and did not reject it. However in one case, the AUD/USD spot and 6-month forward rate, the tests yielded a statistically significant result, providing evidence against the restricting cointegrating vector.

Finally, we evaluated the error correction model for both exchange rates and commonly found that the error correction terms where statistically significant, were of the correct sign but were of an extremely small nature. This meant that convergence to equilibrium would be very slow, with only 5.03%, 1.55%, 0.67%, and 4.13% of discrepancies corrected for each period. Furthermore diagnostically the model suffered from non-normality and in one case heteroscedasticity. To conclude, our results were broadly consistent with the FRUH and market efficiency.

References

Aggarwal, R., B. Lucey, M., et al. (2006). The Forward Exchange Rate Bias Puzzle: Evidence from New Cointegration Tests, Institute for International Integration Studies**:** 1-24.

Baillie, R.T. Lippens, R.E. and P. C. McMahon, (1983) Testing Rational Expectations and Efficiency in the Foreign Exchange Market Econometrica, 51, 3. 553-563

Barnhart, S., W. and A. Szakmary, C. (1991). "Testing the Unbiased Forward Rate Hypothesis:Evidence on Unit Roots, Co-integration; and Stochastic Coefficients." Journal of Financial and Quantitative Analysis **26**(2): 245-267.

Bartolini, L. and L. Giorgianni (2001). "Excess Volatility of Exchange Rates with Unobserved Fundamentals." Review of International Economics **9**(3): 518-530.

Bilson, J. F.O., (1978) "The Monetary Approach to the Exchange Rate: Some Empirical Evidence," IMF Staff Papers 25, 48-75.

BIS (2006). V. Foreign Exchange Markets. BIS 76 Annual Report, Bank of International Settlements**:** 79-97.

Clarida, R.H., and M.P. Taylor (1997) The Term Structure of Forward Exchange Premiums and the Forecastability of Spot Exchange Rates: Correcting the Errors, The Review of Economics and Statistics, 79, 3, 353-361

Daniels, J., P. and D. VanHoose (2001). International Monetary & Financial Economics, South-Western Publishing, Thomson Learning.

DeFAT (2006). Composition of Trade 2005. D. o. F. A. a. Trade, Commonwealth of Australia 2006**:** 21-38.

Dwyer, G., P, Jr. and M. Wallace, S. (1992). "Cointegration and market efficiency." Journal of International Money and Finance **2**: 318-327.

Engle, R.F. and C. W. J. Granger, (1987) Co-Integration and Error Correction: Representation, Estimation, and Testing Econometrica, 55, 2, 251-276

Eichengreen, B., J., J. Tobin, et al. (1995). "Two Cases for Sand in the Wheels of International Finance." The Economic Journal **105**(428): 162-172.

Engle, C. (1996) The forward discount anomaly and the risk premium. A survey of recent

evidence, Journal of Empirical Finance, 3, p. 123-192.

Felmingham, B., S. and S. Leong (2003). Parity Conditions and the Efficiency of the Australian 90 and 180 Day Forward Markets, University of Tasmania, School of Economics**:** 1-42.

Frankel J. A. (1979) On the Mark: a theory of floating exchange rates based on interest rate differentials, "American Economic Review, 69, 601-622.

Frenkel J. A. (1976), A monetary approach to the exchange rate: doctrinal aspects and empirical evidence "Scandinavian Journal of Economics", 78, 200-224.

Granger, C. W. J. and P. Newbold, (1974) Spurious regressions in econometrics, Journal of Econometrics, Elsevier, vol. 2(2) 111-120.

Hakkio, G., S. and M. Rush (1989) Market efficiency and cointegration: an Application to the Sterling and Deutschemark Exchange Markets, Journal of International Money and Finance **8**: 75-88.

Hai, W., N. Mark and Y. Wu, (1997) Understanding Spot and Forward Exchange Rate Regressions Journal of Applied Econometrics 12, (6) 715-734.

Hansen, L. P. and Hodrick, Robert J, (1980) Forward Exchange Rates as Optimal Predictors of Future Spot Rates: An Econometric Analysis, Journal of Political Economy, University of Chicago Press, vol. 88(5) 829-53.

Huang, R.D. (1981) The Monetary Approach to Exchange Rate in an Efficient Foreign Exchange Market: Tests Based on Volatility, Journal of Finance, 36: 31-41,

Isard. P. (1978) Exchange rate determination: a survey of popular views and recent models, Prentice Hall, EngleWood Cliffs, New Jersey.

Johansen, S. (1991) Cointegration in Partial Systems and the Efficiency of Single Equation Analysis, Journal of Econometrics 52, 389-402.

Johansen, S. (1995) Identifying Restrictions of Linear Equations – with applications to simultaneous equations and cointegration". Journal of Econometrics 69, 111-132.

Kahn, G., A. (2000). Global Economic Integration: Opportunities and Challenges, A Summary of the Banks 2000 Economic Symposium. F. R. B. o. K. City, Economic Review**:** 5-15.

Madsen, E. S. (1996). "Inefficiency of foreign exchange markets and expectations: survey evidence." Applied Economics **2**8: 397-403.

Meese, R. A. and K.J. Singleton, (1982) " On Unit Roots and the Empirical Modeling of Exchange Rates," Journal of Finance, 37(4), 1029-35.

Naka, A. and G. Whitney (1995) The unbiased forward rate hypothesis re-examined, Journal of International Money and Finance, 14, 857-867

Peirson, G., R. Brown, et al. (2002). Business Finance, McGraw-Hill Australia Pty Ltd.

Pesaran, H.M. and R. P Smith, The Role of Theory in Econometrics, Journal of Econometrics, 1995, 67 .61-79.

RBA (2004). Survey of Foreign Exchange and Derivatives Markets, Reserve Bank of Australia**:** 1-8.

Sakoulis, G. and E. Zivot (2001). Time-Variation and Structural Change in the Forward Discount: Implications for the Forward Rate Unbiasedness Hypothesis, University of Washington, Department of Economics**:** 1 - 44.

Tease, W. J. (1988). "Speculative Efficiency and the Exchange Rate: some evidence since the float." The Economic Record **64**(184): 2-13.

Vander Kraats R.H. and L.D. Booth (1983) Empirical Tests of the Monetary Approach to Exchange Rate Determination, Journal of International Money and Finance, 2, 255-278.

Zivot, E. (1997). Cointegration and Forward and Spot Exchange Rate Regressions, University of Washington, Department of Economics**:** 1 - 36.

Wadhwani, S. (1987) THE EXCHANGE RATE AND THE MPC: WHAT CAN WE DO?, www.bankofengland.co.uk/publications/speeches/2000/speech88.pdf

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